ENVIRONMENTAL QUALITY POSSIBILITIES (E.Q.P.):
A PROCEDURE FOR EVALUATING ECONOMIC/ENVIRONMENTAL TRADE-OFFS

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ABSTRACT

Two co-equal objectives exist in water quality planning: national economic development--that is increased production of goods and services--and environmental quality--the enhancement of physical, ecological, and aesthetic characteristics. This paper presents a methodology for evaluating possible trade-offs between these two objectives and demonstrates its feasibility. Values of environmental quality are first quantified using the S.Q.P.I. (System for Quantified Planning Inquiry) methodology. The resulting environmental goal structure is then linked to a separable mathematical programming procedure. This produces a production possibilities curve which illustrates the impacts of alternative choices on the two objectives. An example is presented to illustrate the application of the procedures.

Key words: cost-benefit analysis; trade-offs; environmental quality; separable programming; multiple objective planning; water quality.

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ENVIRONMENTAL QUALITY POSSIBILITIES (E.Q.P.): A PROCEDURE FOR EVALUATING ECONOMIC/ENVIRONMENTAL TRADE-OFFS

INTRODUCTION

The purpose of this paper is to demonstrate the feasibility of a methodology incorporating the System for Quantified Planning Inquiry (S.Q.P.I.) methodology with mathematical programming. The new methodology, termed Environmental Quality Possibilities (E.Q.P.), produces a production possibilities curve illustrating possible trade-offs between economic growth and environmental quality.

This paper will describe the development of a production possibilities curve for a simple hypothetical example by means of a separable programming model.

BACKGROUND

Benefit Cost Analysis

The benefit-cost ratio has been the traditional tool for determining the desirability of various projects and policies proposed by the government. A project is determined feasible if anticipated monetary benefits exceed the expected monetary costs or, in other words, if the benefit-cost ratio is greater than one.

Benefit-cost analysis is most effective when all the benefits and costs of a project or policy implementation can be estimated and easily

formulated in monetary terms. Unfortunately, this is frequently not the case, and estimating expected costs and benefits is a time consuming and often inexact process (see, for instance, Krutilla, and Fisher 1975). Furthermore, because of the limitations of the traditional approach, the majority of environmental benefits and costs have not been formulated. Environmental-economic trade-off analyses have traditionally involved only estimates of the costs involved in meeting various physical and biological standards related to improving environmental quality. By concentrating only on selected physical standards and not perceived environmental quality, only part of the information necessary and useful for policy analysis is developed. It would be helpful to policy makers if more precise information regarding the various trade-offs between economic costs and perceived environmental quality could be provided.

New methodologies are available that provide needed information to the decision maker through tools for integrating the "hard to quantify" benefits and costs into the process of evaluating the economic feasibility of a project. One such methodology is the System for Quantified Planning Inquiry (Arthur, Bowes, and Gum, 1976) which is based on the work of the Technical Committee of the Water Resource Centers of the Thirteen Western States (The Technical Committee, 1974). S.Q.P.I. allows for the consideration of multiple objectives and is very effective in evaluating non-monetary objectives such as environmental quality. The Environmental Quality Possibilities utilizes S.Q.P.I. estimates of perceived environmental quality for developing its production possibilities curve.

THE S.Q.P.I. GOAL STRUCTURE

Before quantitative values of environmental quality can be obtained from the S.Q.P.I. System, the subcategories or "subgoals" of environmental quality must be identified (figure 1). Several rules were followed with respect to structuring these goals and subgoals (Gum, 1974), including:

1. the structure should be dendritic, moving from the general goals, through the subgoals to specific social indicators;

2. the structure of goals, subgoals, and social indicators should be nonoverlapping in nature, but each may appear in more than one place in the overall structure;

3. each of the individual subgoals listed in any one category should be independent within that category;

4. all the possible components of a goal or subgoal should be listed; and

5. because estimates are that individuals can judge simultaneously only six or seven items (Schimpeler, 1967, p. 146), the maximum number of subgoals in any one category should be limited to six or seven.

Once a goal structure has been defined, the goals can be quantified. Information in the system must be passed from a set of "social indicators"--measurable quantities related to the lowest level goals--to the goals in the higher levels (figure 1). A power function is used to aggregate subgoals into high level goals. The value of a goal will reflect the value of social indicators which influence the goals and the relative importance of subgoals comprising a goal. The process of selecting and evaluating environmental goals will be illustrated for a hypothetical water quality problem.

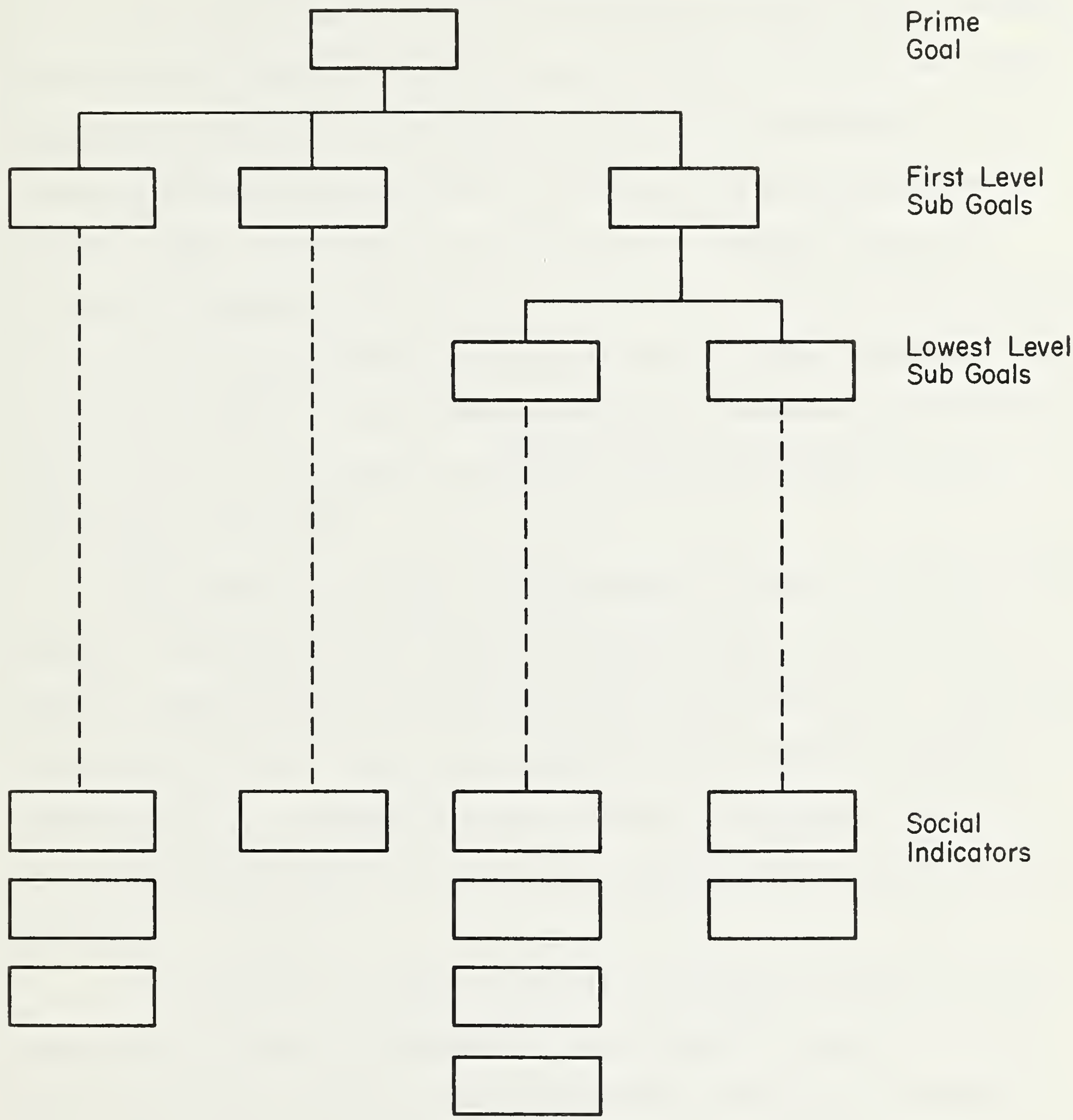
Prime Goal
First Level Sub Goals
Lowest Level Sub Goals
Social Indicators

Figure 1.

Development of Water Quality Goal Structure

A structure designed to aid in evaluating the water quality impacts of conservation practices was constructed as a part of Project 42-308 "The Integration of Society's Environmental Quality Goals and Preferences into the Principles and Standards Resource Evaluation and Planning Framework" (Oswald, 1978). According to the Principles and Standards (WRC 1974), alternative management plans are to be evaluated by their successful achievement of goals which have been previously defined in terms of specific output or effects (figure 2). Water quality parameters, through which desired outputs or effects can be described easily, served as the basis for goal achievement evaluation.

The highest level goal in the structure in figure 2 is environmental quality. The first level of subgoals includes natural resource quality, cultural resource quality, recreation quality, and human health. The study concentrates on water quality as determined by the perceived importance of floating objects in the water, clarity, and odor. The clarity factor is further subdivided since it is affected by algae and the sediment or mud in the water.

This structure of goals can be used to assess the impacts on water quality of a proposed plan component. As diagrammed in figure 2, a plan involving a single component such as streambank protection, for example, would lower concentrations of suspended solids, total solids, and measures of turbidity and color, thereby affecting water quality and eventually environmental quality.

How well alternative plans for the development of water and related land resources achieve or improve environmental quality or relevant subgoals

GOAL STRUCTURE

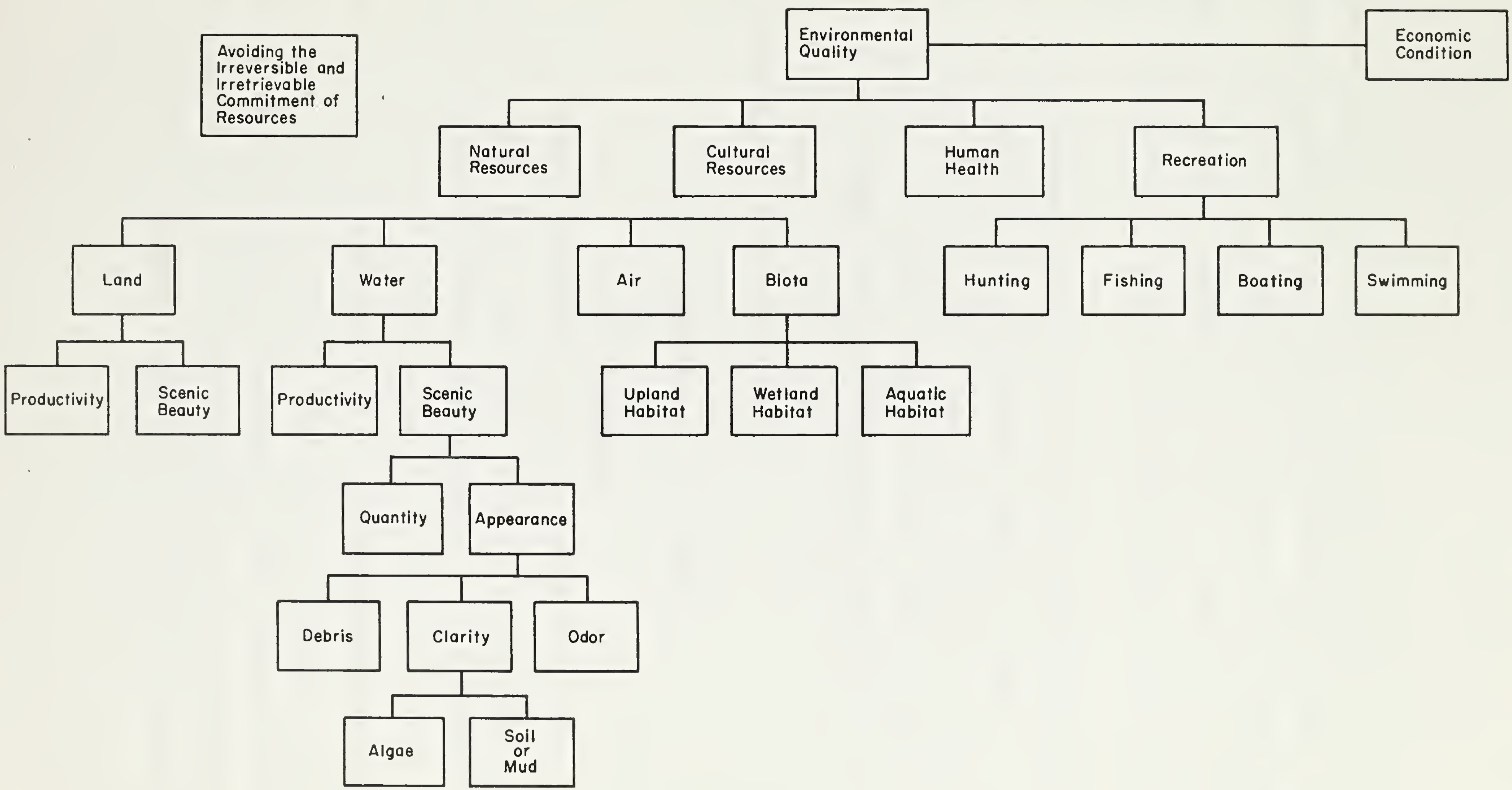

Figure 2

can also be determined. For example, it is possible to quantify the results of streambank protection (acres of streambank protection = decrease in turbidity of X milligrams per liter) and then mathematically link this measure to subgoals at each level (see Gum et al., 1976). Through such a procedure, alternative plans could be ranked as to how they achieve overall environmental quality.

After obtaining information on the impacts of alternative water pollution control activities on environmental quality, the costs of achieving alternative levels of quality must be measured. The remaining sections of the paper discuss the process of linking the S.Q.P.I. goal structure to a mathematical programming procedure which maximizes environmental quality subject to restraints on levels of profits. The result of the linkage is Environmental Quality Possibilities (E.Q.P.).

LINEAR PROGRAMMING - S.Q.P.I. LINKAGE

Linear Programming

Many problems in economics are basically concerned with the allocation of limited resources--land, labor, space, time--in order to maximize some measure of performance or minimize some measure of cost. Mathematical programming refers to the mathematical techniques for solving such allocation problems. Linear programming defines a situation where the performance or cost measure is a linear function of the controllable variables. Linear programming involves maximizing or minimizing a linear function of several variables subject to a set of linear constraints. Linear programming differs from the classical optimization problem because it introduces inequalities

as constraints. Thus, the usual techniques of calculus cannot be used.

Separable Programming

Separable programming is a process by which a nonlinear objective function is approximated by a piecewise linear function (McMillan, 1970). The process is not constrained to second degree functions or to linear constraints. The objective function must be concave, however, if it is to be maximized and convex, if it is to be minimized. Separable programming reduces a nonlinear programming problem to a series of linear approximations which can then be solved by linear programming.

The reduction of a log function to a stepwise linear function is the key to combining the S.Q.P.I. goal structure, with its power function formulation of goal achievement, and traditional linear programming analysis of least cost methods of meeting pollution standards. The function $f(x) = \log x$ is plotted in figure 3 along with a five segment approximation.

From the linear to the log example above, the range of values can be divided into segments. Then the nonlinear log function can be approximated by the linear functions which connect the points on the log function. To improve the accuracy of the results, it would be necessary to use a large number of line segments. For purposes of illustration, however, five segments will suffice.

The following formula can be used to determine the equation for each of the five lines, each with its own slope and intercept.

$$f(x) = f_k + \frac{f_{k+1} - f_k}{x_{h+1} - x_k}(x - x_k)$$

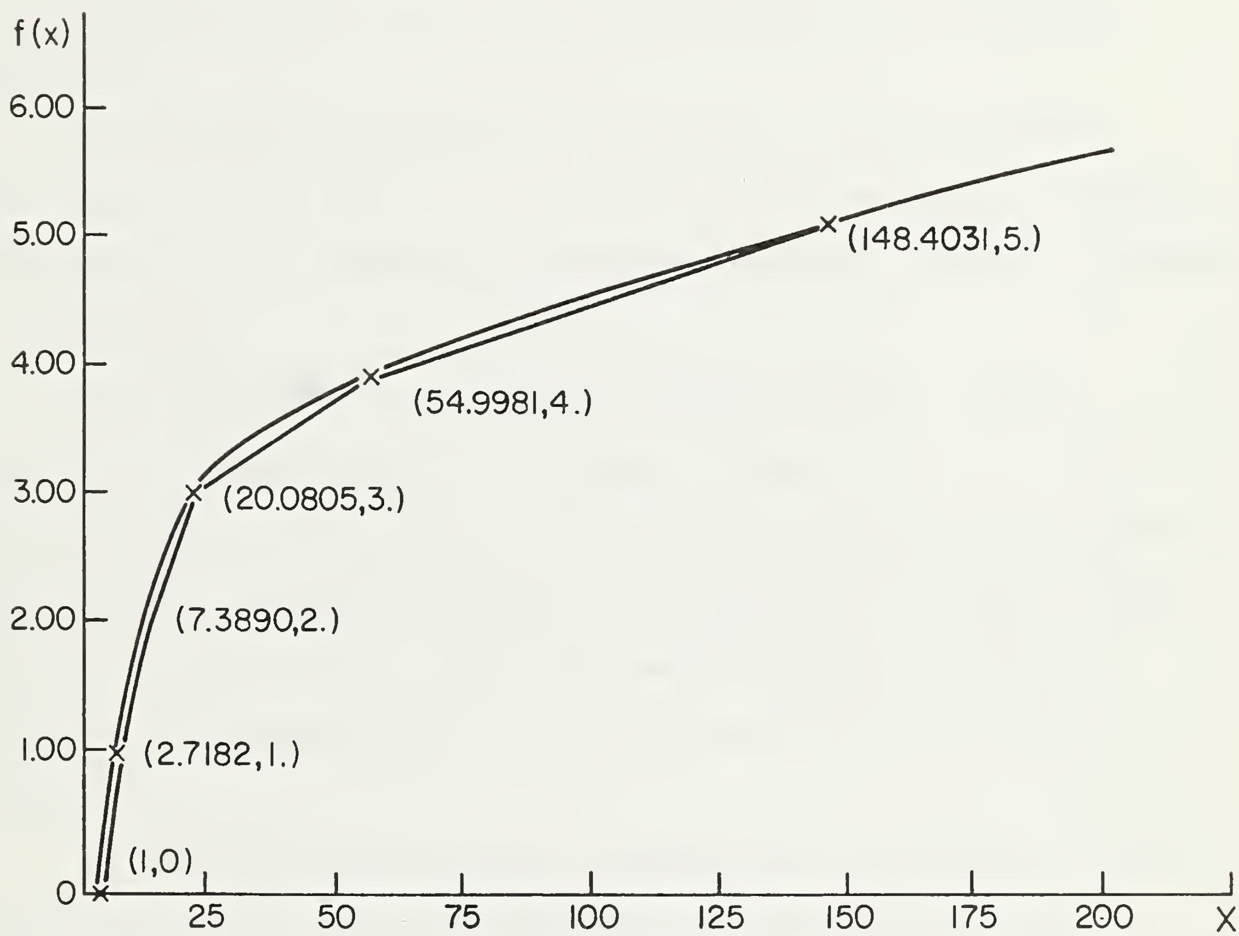
f (x)
6.00
5.00
4.00
3.00
2.00
1.00
0
(148.4031,5.)
(54.9981,4.)
(20.0805,3.)
(7.3890,2.)
(2.7182,1.)
(1,0)
25
50
75
100
125
150
175
200
X

Figure 3

The subscripts k and k + 1 refer to consecutive points among our points. The line segment connecting the two points (2.7182,1) and (7.3890,2), for example when substituted into the formula, becomes

$$f(X_1) = 1 + \frac{(2 - 1)}{(7.3890 - 2.7182)} (x - 2.7182) = .2140x - .5187$$

All equations for all other line segments in the example can be similarly determined.

With the linear approximations established, a linear programming model can be set up. First, it is necessary to reduce the separate equations for the different line segments into one single equation. This is accomplished by the following:

$$\text{approximate log } X = 1 + .2140X_2 + .0787X_3 + .0286X_4 + .0107X_5$$

Where X_2 is the number of units the variable X is above 2.7182 (e). X_2 is also constrained to be less than 4.6708, which is the number of units between the first point on the approximation 2.7182,1 and the next 7.3890,2. (Other points on the function f(x) = log x could have been chosen as appropriate points (in the approximation). X_3, X_4, and X_5 are defined in a similar manner.

To approximate the log X as a function of X, X is divided into the appropriate X_2, X_3, X_4, X_5, the approximation slopes multiplied times the X_2, X_3, X_4, X_5 variables, and the results summed. To remove the possibility of negative logs, it is assumed that X_1 will always be 2.7182. Therefore, the 1 appears in the equation to approximate the logs.

How can this (approximation of a log function by a series of straight line segments) be used to combine S.Q.P.I. goals with traditional linear programming analysis? Consider maximizing a S.Q.P.I. index of environmental

quality. The S.Q.P.I. index would be a power function of a set of environmental variables. For the example, the S.Q.P.I. formulation of two components of an E.Q. index, X, Y, would be

$$\text{E.Q. Index} = X^{PW_x} Y^{PW_y}$$

where PW_x and PW_y are preference weights for achievement of subgoal X and subgoal Y. Assume the P values are .3 and .7 respectively. Then

$$\text{E.Q. Index} = X^{.3}Y^{.7}$$

and

$$\log \text{E.Q. Index} = .3 \log X + .7 \log Y.$$

By use of our estimations:

approximate log E.Q. Index = .3 (approximate log X) + .7 (approximate log Y)

which is now a linear equation in terms of X_2, X_3, X_4, X_5, Y_2, Y_3, Y_4, Y_5, and can be used as the objective function of an environmental quality maximization linear programming problem. The following example illustrates such a problem.

Linear Programming Model Example

This section will demonstrate how to use a linear approximation of a power function to establish an environmental index for a linear programming model. By listing various profits or returns to the firm with the appropriate figure in the environmental index, a production possibilities table can be derived and graphed which emphasizes the trade-offs that exist between environmental quality and profits. Profits are used as a proxy for the value of economic production.

First, a linear programming Environmental Quality maximization model, as developed in the previous section, is constructed. The objective function (Cobb-Douglas) is

$$\text{MAX } E = X^a Y^b.$$

E will yield the environmental quality index which is maximized. Assume X represents water quality and Y represents land quality. E is maximized when the level of water quality and land quality are maximized. Initially, the levels of X and Y will be the result of a particular farming practices. Then, alternative practices are introduced which increase or decrease the levels of X and Y as well as affect costs of the operation. This will make it possible to construct a production possibilities table and curve.

The entire tableau for the problem includes the objective function, linear estimation of the nonlinear function f(x) = log x and the basic production and conservation activities.

$$\text{MAX } .3 \text{ FLOG } X + .7 \text{ FLOG } Y$$

The following constraints are necessary

$$\text{FLOG } X - .2140X_2 - .0787X_3 - .0286X_4 - .0107X_5 = 1$$

which is the approximation for the log X. Furthermore the limits for the various X's according to the broken line segments must be set

$$X_2 \leq 4.6708$$

$$X_3 \leq 12.6965$$

$$X_4 \leq 34.9126$$

$$X_5 \leq 93.4050$$

$$X - X_2 - X_3 - X_4 - X_5 = 2.7182$$

The same steps can be taken for Y.

The profit equation that establishes the revenue that can be expected from farming practice FP_1 and FP_2 and the costs that are associated with the conservation practices CP_1 and CP_2 is

Profit $\quad 50FP_1 + 30FP_2 - 10CP_1 - 50CP_2 = \$5000$

FP_1 could represent the production of corn while FP_2 could represent the production of wheat. They could also each represent some particular technique or policy in the production of an identical product. FP_1 and FP_2 add revenue to the entrepreneur's budget while CP_1 and CP_2 are regarded as costs that must be paid to adopt a particular conservation practice. They may improve the environment, but they subtract from overall profits. CP_2 is considerably more expensive than CP_1.

Land $\quad 1FP_1 + 1FP_2 \leq 100$

Treatment $\quad 1CP_1 + 1CP_2 \leq 100$

The land constraint, as well as the treatment constraint, establish the limits on the amount of land available for the two farming and conservation practices. There are 100 acres of land on which either FP_1, FP_2 or some combination of the two can be employed. The same is true for CP_1 and CP_2.

Sediment $\quad X + .1FP_1 + .5FP_2 - .1CP_1 - .2CP_2 = 100$

Erosion $\quad Y + .2FP_1 + .1FP_2 \qquad - .3CP_2 = 100$

The sediment and erosion constraints estimate the amount of sediment and erosion generated or eliminated through the different farming and conservation practices. In this simple example the water quality variable X

becomes 100 less the sediment value (e.g. if sediment were to equal 100 the water quality would be zero). The sediment equation states that farming practice FP_1 produces .1 unit of sediment per acre while FP_2 produces .5 unit of sediment. Conservation practice CP_1 reduces sediment .1 unit per acre and CP_2 reduces sediment .2 unit per acre. A similar description holds true for erosion in the land quality equation.

Once the constraints are listed, a computerized solution algorithm is employed to compute the maximum environmental index subject to the constraints. The result from the example is that 100 acres of land are appropriated to the use of FP_1 with an environmental index of 73.6. By reducing the profit constraint to $4000, $3000, $2000, $1000 and $500, the environmental index is improved as seen below. (The environmental index is derived by taking the solution value in log form and calculating its real value.)

Profit	$5000	$4000	$3000	$2000	$1000	$500
FP_1	100	100	100	100	100	100
FP_2	0	0	0	0	0	0
CP_1	0	0	0	0	0	0
CP_2	0	20	40	60	80	90
Environmental Index	73.6	78.0	82.7	87.6	92.8	95.5

As the profit is reduced from $5000 to $4000 this introduces an allocation of 20 acres to CP_1. As a result of this shift, the environmental index rises to 78.0. Further profit reductions cause additional changes until the highest environmental index of 95.5 is reached when only $500 profit is earned. The environmental index is improved only at a loss of profit which illustrates the trade-offs between the two. A graph of these trade-offs, the standard production possibilities curve, is shown in figure 4.

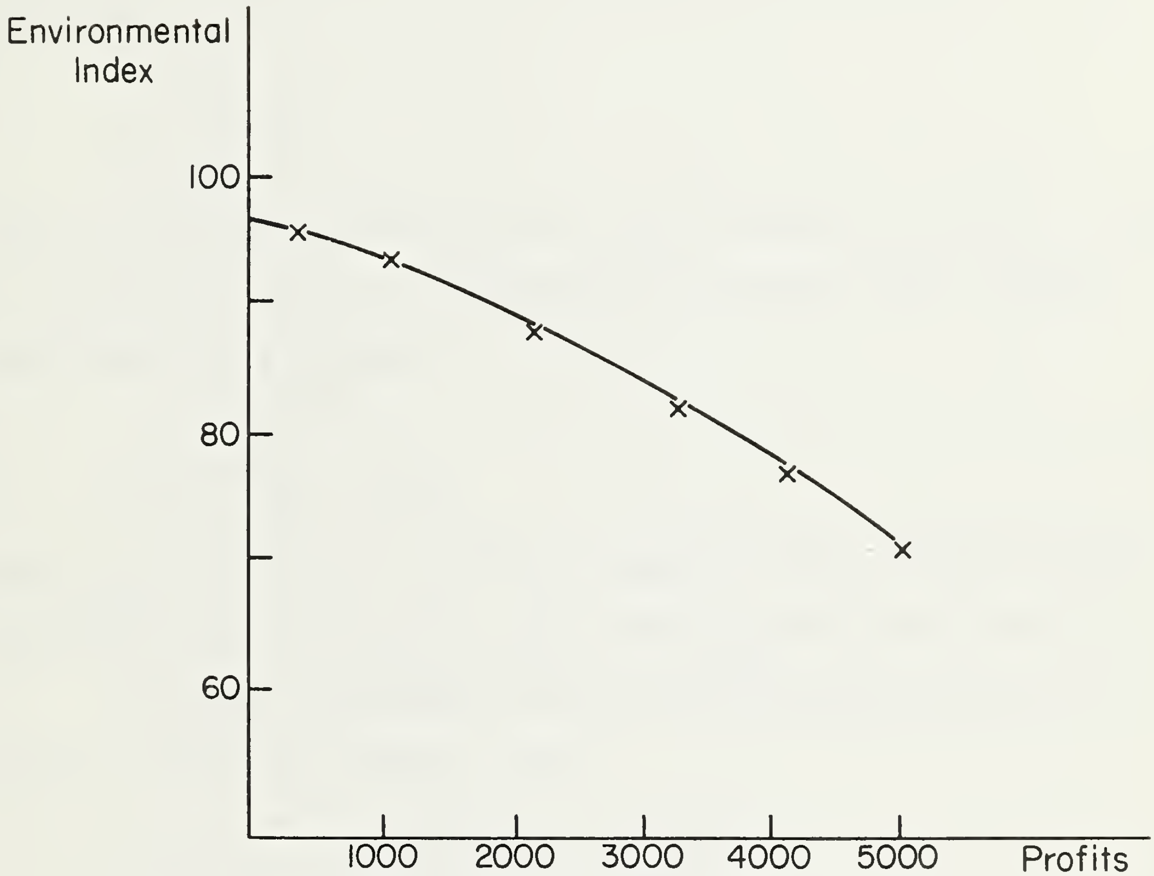
Environmental Index
100
80
60
1000
2000
3000
4000
5000
Profits

Figure 4

Alternative Formulation

In the preceding section, separable programming is used to approximate a nonlinear function by a piecewise linear function. The first step in the formulation of a piecewise linear function is to express X_j as a sum of variables

$$X_j = \sum_{k=1}^{n_j} X_{jk}$$

where X_{jk} represents the level used of the k^{th} segment of activity j, and n_j is the number of segments. Consequently the piecewise linear function approximating $f_j(X_j)$ becomes

$$f_j(X_j) = \sum_{k=1}^{n_j} S_{jk} X_{jh}$$

where S_{jk} is the slope of the k^{th} line segment. This is subject to the constraints that each X_{jk} has an upper bound U_{jk} and a lower bound zero.

The linear programming formulation for the problem would be as follows (Hillier and Lieberman, 1974).

$$\text{Maximize } Z = \sum_{j=1}^{n} \left(\sum_{k=1}^{n_j} S_{jk} X_{jk} \right)$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} \left(\sum_{k=1}^{n_j} x_{jk} \right) \leq b_i \qquad \text{for } i = 1, 2, \ldots . m$$

$$X_{jk} \leq U_{jk} \qquad \text{for } k = 1, 2, \ldots n_j \text{ and } j = 1, 2, \ldots n$$

$$\text{and} \quad X_{jk} \geq 0 \qquad \text{for } k = 1, 2, \ldots n_j \text{ and } j = 1, 2, \ldots n$$

The above formulation maximizes Z as the sum of the contributions from the respective line segments. On effect, this can be interpreted as moving from one break point to the next, and then just a proportion up the last segment x_{jk}/U_{jk}. Consequently an alternative formulation would be to interpolate along that final segment a proportion $W_{jk} = x_{jk}/U_{jk}$. Consequently, $f_j(X_j)$ would be formulated

$$f_j(x_j) = \sum_{k=1}^{n_j} P_{jk} W_{jk}$$

subject to $\displaystyle\sum_{k=1}^{n_j} W_{jk} \leq 1$

and $\quad W_{jk} \geq 0 \qquad$ for $k = 1, 2, \ldots n_j$

The W_{jk} variables are interpreted as interpolation weights. The interpolation must be between the two end points of a single line segment.

For this alternative formulation, the linear programming formulation is

$$\text{Maximize} \quad Z = \sum_{j=1}^{n} \sum_{k=1}^{n_j} P_{jk} W_{jk}$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} \left[\sum_{k=1}^{n_j} \left(\sum_{l=1}^{k} U_{jl} \right) W_{jk} \right] \quad b_i \text{ for } i = 1, 2, \ldots m$$

$$\sum_{k=1}^{n_j} W_{jk} \leq 1 \qquad \text{for } j = 1, 2, \ldots n$$

$$W_{jk} \geq 0 \qquad \text{for } h = 1, 2, \ldots n_j$$

$$\text{and } j = 1, 2, \ldots n.$$

This alternative formulation has only (m + n) constraints as contrasted with the first formulation that has $(m + \Sigma_{j=1}^{n} n_j)$ constraints. The fewer constraints substantially reduces computational time.

The entire tableau for the problem establishing an environmental index using the alternative formulation can now be constructed. The objective function becomes

$$\text{MAX } .3(1W_1 + 2W_2 + 3W_3 + 4W_4 + 5W_5) + .7(1Z_1 + 2Z_2 + 3Z_3 + 4Z_4 + 5Z_5)$$

The coefficients in the objective function are the natural logs of the various values of e. For example, 3 is the natural log of 20.0805 (3e).

Then the following constraints must be added

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 \leq 1$$

$$X - 2.7182W_1 - 7.3890W_2 - 20.0805W_3 - 54.9981W_4 - 148.4031W_5 = 0$$

In the above constraint, the values of 1e, 2e, 3e, etc., are the respective coefficients.

$$Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 \leq 1$$

$$Y - 2.7182Z_1 - 7.3890Z_2 - 20.0805Z_3 - 54.9981Z_4 - 148.4031Z_5 = 0$$

The profit, land, treatment, sediment, and erosion constraints remain the same.

Through this formulation the number of constraints is reduced to nine. An additional advantage is that increasing the number of line segments in the approximation adds only additional columns to the tableau. The number of rows remain constant.

SUMMARY AND CONCLUSIONS

Through a linear programming model, an environmental index was constructed which then made it possible to establish a production possibilities

table and curve illustrating the trade-offs that exist between environmental quality and production. In this example, improvement in the quality of the environment can be achieved only at the cost of production. Furthermore, because of the separable programming model, it is possible to measure the trade-offs that exist and to make better informed decisions on the choices available to society.

This new methodology provides more information than is provided by the traditional benefit-cost analysis. Policy makers are able to judge several different alternatives at the same time, rather than independently judge one individual project. More attention can be given specifically to environmental benefits and costs which must frequently be passed over in traditional benefit-cost analysis.

The example presented in this paper demonstrates the feasibility of combining a TECHCOM-S.Q.P.I. approach with linear programming models. The practical feasibility of using a structure of environmental subgoals and much more complicated set of farming and conservation practices remains to be demonstrated. The authors are now working in such a project.

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